9 Quadratic Equations and Functions

COMMON CORE

Lesson	
9.1	CC.9-12.F.BF.3
9.2	CC.9-12.F.IF.7a*
9.3	CC.9-12.F.IF.7a*
9.4	CC.9-12.A.REI.4b
9.5	CC.9-12.A.REI.4b
9.6	CC.9-12.A.REI.4b
9.7	CC.9-12.A.REI.11*
9.8	CC.9-12.A.CED.2*
9.9	CC.9-12.F.IF.4*

Before

Previously, you learned the following skills, which you'll use in this chapter: reflecting points in a line and finding square roots.

Prerequisite Skills

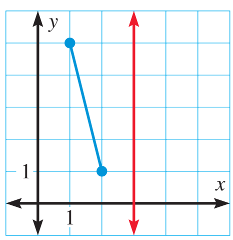
VOCABULARY CHECK

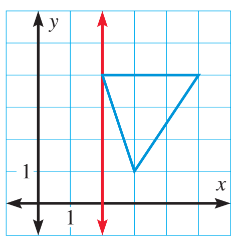
Copy and complete the statement.

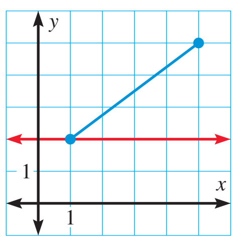
1. The x-coordinate of a point where a graph crosses the x-axis is a(n) _?_.
2. A(n) _?_ is a function of the form $y = a \cdot b^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$.

SKILLS CHECK

Draw the blue figure. Then draw its image after a reflection in the red line.

3. 

4. 

5. 

Evaluate the expression.

6. $\sqrt{81}$ 7. $-\sqrt{25}$ 8. $\sqrt{1}$ 9. $\pm\sqrt{64}$

Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

1. **Graphing quadratic functions**
2. **Solving quadratic equations**
3. **Comparing linear, exponential, and quadratic models**

KEY VOCABULARY

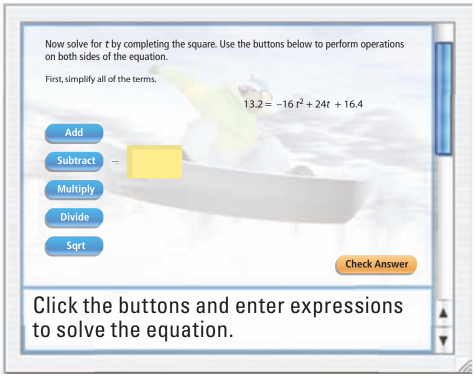
- quadratic function
- parabola
- parent quadratic function
- vertex
- axis of symmetry
- minimum value
- maximum value
- quadratic equation
- completing the square
- quadratic formula

Why?

You can use a quadratic model for real-world situations involving vertical motion. For example, you can write and solve a quadratic equation to find the time a snowboarder is in the air during a jump.

Animated Algebra

The animation illustrated below helps you answer a question from this chapter: How many seconds is the snowboarder in the air during a jump?



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9.1 Graph $y = ax^2 + c$

Before	You graphed linear and exponential functions.
Now	You will graph simple quadratic functions.
Why?	So you can solve a problem involving an antenna, as in Ex. 40.

Key Vocabulary
- **quadratic function**
- **parabola**
- **parent quadratic function**
- **vertex**
- **axis of symmetry**

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

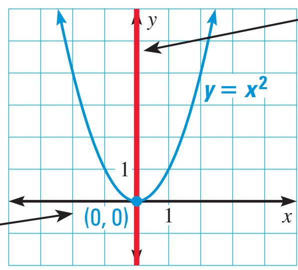
A **quadratic function** is a nonlinear function that can be written in the **standard form** $y = ax^2 + bx + c$ where $a \neq 0$. Every quadratic function has a U-shaped graph called a **parabola**. In this lesson, you will graph quadratic functions where $b = 0$.

KEY CONCEPT — *For Your Notebook*

Parent Quadratic Function

The most basic quadratic function in the family of quadratic functions, called the **parent quadratic function**, is $y = x^2$. The graph of $y = x^2$ is shown below.

The lowest or highest point on a parabola is the **vertex**. The vertex of the graph of $y = x^2$ is (0, 0).



The line that passes through the vertex and divides the parabola into two symmetric parts is called the **axis of symmetry**. The axis of symmetry for the graph of $y = x^2$ is the y-axis, $x = 0$.

EXAMPLE 1 Graph $y = ax^2$ where $|a| > 1$

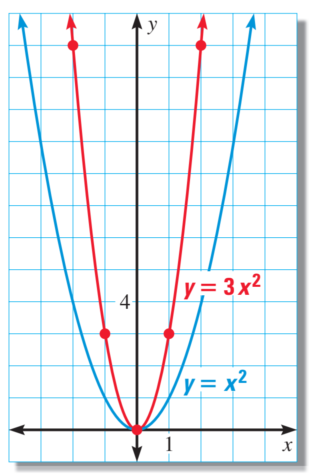
STEP 1 **Make** a table of values for $y = 3x^2$.

x	−2	−1	0	1	2
y	12	3	0	3	12

STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = 3x^2$ and $y = x^2$. Both graphs open up and have the same vertex, (0, 0), and axis of symmetry, $x = 0$. The graph of $y = 3x^2$ is narrower than the graph of $y = x^2$ because the graph of $y = 3x^2$ is a vertical stretch (by a factor of 3) of the graph of $y = x^2$.



DESCRIBE A FUNCTION
A quadratic function has an unbroken graph, so the function is continuous.

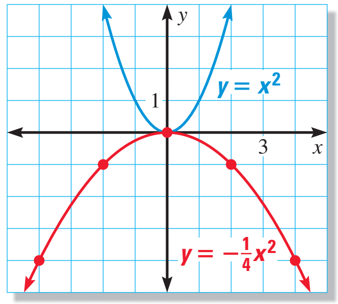
EXAMPLE 2 Graph $y = ax^2$ where $|a| < 1$

Graph $y = -\frac{1}{4}x^2$. Compare the graph with the graph of $y = x^2$.

MAKE A TABLE
To make the calculations easier, choose values of x that are multiples of 2.

STEP 1 **Make** a table of values for $y = -\frac{1}{4}x^2$.

x	−4	−2	0	2	4
y	−4	−1	0	−1	−4



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = -\frac{1}{4}x^2$ and $y = x^2$. Both graphs have the same vertex (0, 0), and the same axis of symmetry, $x = 0$. However, the graph of $y = -\frac{1}{4}x^2$ is wider than the graph of $y = x^2$ and it opens down. This is because the graph of $y = -\frac{1}{4}x^2$ is a vertical shrink $\left(\text{by a factor of } \frac{1}{4}\right)$ with a reflection in the x-axis of the graph of $y = x^2$.

GRAPHING QUADRATIC FUNCTIONS Examples 1 and 2 suggest the following general result: a parabola opens up when the coefficient of x^2 is positive and opens down when the coefficient of x^2 is negative.

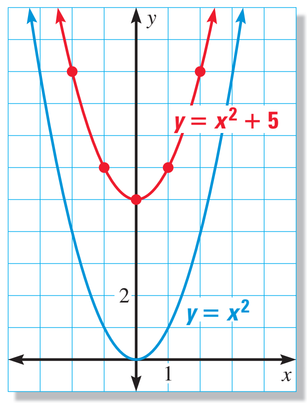
EXAMPLE 3 Graph $y = x^2 + c$

Graph $y = x^2 + 5$. Compare the graph with the graph of $y = x^2$.

ANALYZE RATE OF CHANGE
Notice that for a quadratic function, the rate of change in y with respect to x is *not* constant as it is for a linear function. For instance, $\frac{6-9}{-1-(-2)} = -3$, while $\frac{5-6}{0-(-1)} = -1$.

STEP 1 **Make** a table of values for $y = x^2 + 5$.

x	−2	−1	0	1	2
y	9	6	5	6	9



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = x^2 + 5$ and $y = x^2$. Both graphs open up and have the same axis of symmetry, $x = 0$. However, the vertex of the graph of $y = x^2 + 5$, (0, 5), is different than the vertex of the graph of $y = x^2$, (0, 0), because the graph of $y = x^2 + 5$ is a vertical translation (of 5 units up) of the graph of $y = x^2$.

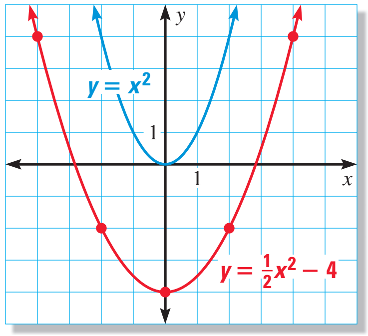
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the function. Compare the graph with the graph of $y = x^2$.

1. $y = -4x^2$ **2.** $y = \frac{1}{3}x^2$ **3.** $y = x^2 + 2$

EXAMPLE 4 Graph $y = ax^2 + c$

Graph $y = \frac{1}{2}x^2 - 4$. Compare the graph with the graph of $y = x^2$.



STEP 1 **Make** a table of values for $y = \frac{1}{2}x^2 - 4$.

x	−4	−2	0	2	4
y	4	−2	−4	−2	4

STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = \frac{1}{2}x^2 - 4$ and $y = x^2$. Both graphs open up and have the same axis of symmetry, $x = 0$. However, the graph of $y = \frac{1}{2}x^2 - 4$ is wider and has a lower vertex than the graph of $y = x^2$ because the graph of $y = \frac{1}{2}x^2 - 4$ is a vertical shrink and a vertical translation of the graph of $y = x^2$.

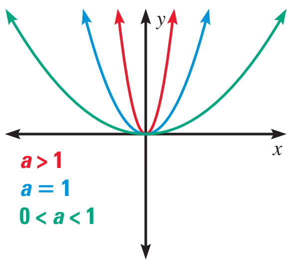
✓ GUIDED PRACTICE for Example 4

Graph the function. Compare the graph with the graph of $y = x^2$.

4. $y = 3x^2 - 6$ **5.** $y = -5x^2 + 1$ **6.** $y = \frac{3}{4}x^2 - 2$

KEY CONCEPT *For Your Notebook*

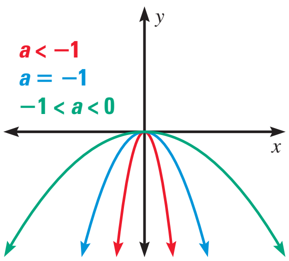
$y = ax^2, a > 0$



Compared with the graph of $y = x^2$, the graph of $y = ax^2$ is:

- a vertical stretch if $a > 1$,
- a vertical shrink if $0 < a < 1$.

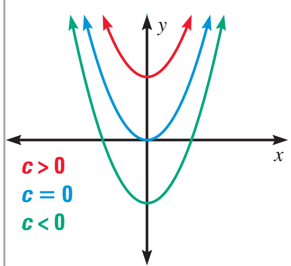
$y = ax^2, a < 0$



Compared with the graph of $y = x^2$, the graph of $y = ax^2$ is:

- a vertical stretch with a reflection in the x-axis if $a < -1$,
- a vertical shrink with a reflection in the x-axis if $-1 < a < 0$.

$y = x^2 + c$



Compared with the graph of $y = x^2$, the graph of $y = x^2 + c$ is:

- an upward vertical translation if $c > 0$,
- a downward vertical translation if $c < 0$.

EXAMPLE 5 Standardized Test Practice

How would the graph of the function $y = x^2 + 6$ be affected if the function were changed to $y = x^2 + 2$?

Ⓐ The graph would shift 2 units up.

Ⓑ The graph would shift 4 units up.

Ⓒ The graph would shift 4 units down.

Ⓓ The graph would shift 4 units to the left.

ELIMINATE CHOICES
You can eliminate choice D because changing the value of c in a function of the form $y = x^2 + c$ translates the graph up or down.

Solution

The vertex of the graph of $y = x^2 + 6$ is 6 units above the origin, or (0, 6). The vertex of the graph of $y = x^2 + 2$ is 2 units above the origin, or (0, 2). Moving the vertex from (0, 6) to (0, 2) translates the graph 4 units down.

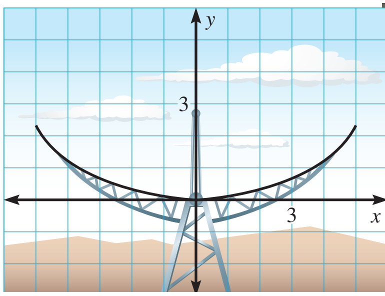
▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 6 Use a graph

SOLAR ENERGY A solar trough has a reflective parabolic surface that is used to collect solar energy. The sun's rays are reflected from the surface toward a pipe that carries water. The heated water produces steam that is used to produce electricity.

ANALYZE A GRAPH
You can use the graph to estimate the range of the function. The lowest point is at the origin. The two highest points are just above the line $y = 2$. So, an estimate of the range is $0 \le y \le 2$.

The graph of the function $y = 0.09x^2$ models the cross section of the reflective surface where x and y are measured in meters. Use the graph to find the domain and range of the function in this situation.



Solution

STEP 1 **Find** the domain. In the graph, the reflective surface extends 5 meters on either side of the origin. So, the domain is $-5 \le x \le 5$.

STEP 2 **Find** the range using the fact that the lowest point on the reflective surface is (0, 0) and the highest point occurs at $x = 5$ or $x = -5$.

$y = 0.09(5)^2 = 2.25$ **Substitute 5 for *x*. Then simplify.**

The range is $0 \le y \le 2.25$.

✓ GUIDED PRACTICE for Examples 5 and 6

7. *Describe* how the graph of the function $y = x^2 + 2$ would be affected if the function were changed to $y = x^2 - 2$.

8. **WHAT IF?** In Example 6, suppose the reflective surface extends just 4 meters on either side of the origin. Find the domain and range of the function in this situation.

9.1 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 7 and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 22, 33, 43, and 44

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Every quadratic function has a U-shaped graph called a(n) __?__.

2. ★ **WRITING** *Explain* how you can tell whether the graph of a quadratic function opens up or down.

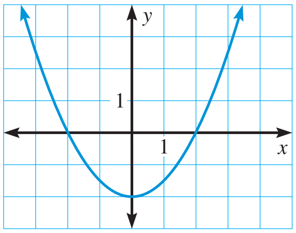
MATCHING **Match the quadratic function with its graph.**

3. $y = \frac{1}{2}x^2 - 4$

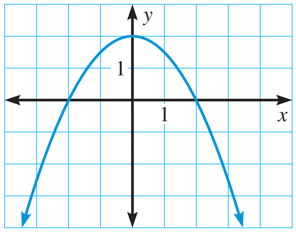
4. $y = \frac{1}{2}x^2 - 2$

5. $y = -\frac{1}{2}x^2 + 2$

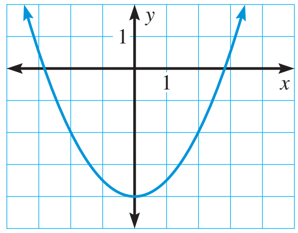
A.



B.



C.



EXAMPLES 1, 2, and 3 for Exs. 6–23

GRAPHING QUADRATIC FUNCTIONS **Graph the function. Compare the graph with the graph of $y = x^2$.**

6. $y = 8x^2$

7. $y = -2x^2$

8. $y = -3x^2$

9. $y = 5x^2$

10. $y = \frac{11}{2}x^2$

11. $y = \frac{2}{3}x^2$

12. $y = -\frac{3}{4}x^2$

13. $y = -\frac{1}{9}x^2$

14. $y = \frac{3}{8}x^2$

15. $y = -\frac{1}{5}x^2$

16. $y = x^2 - 7$

17. $y = x^2 + 9$

18. $y = x^2 + 6$

19. $y = x^2 - 4$

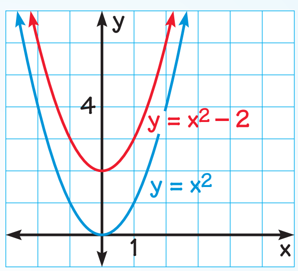
20. $y = x^2 - 1$

21. $y = x^2 + \frac{7}{4}$

22. ★ **MULTIPLE CHOICE** What is the vertex of the graph of the function $y = -\frac{3}{4}x^2 + 7$?

Ⓐ (−7, 0) Ⓑ (0, −7) Ⓒ (0, 7) Ⓓ (7, 0)

23. **ERROR ANALYSIS** *Describe* and correct the error in drawing and comparing the graphs of $y = x^2$ and $y = x^2 - 2$.



Both graphs open up and have the same axis of symmetry. However, the vertex of the graph of $y = x^2 - 2$, (0, 2), is 2 units above the vertex of the graph of $y = x^2$, (0, 0).

EXAMPLE 4
for Exs. 24–32

GRAPHING QUADRATIC FUNCTIONS **Graph the function. Compare the graph with the graph of $y = x^2$.**

24. $y = 7x^2 + 7$

25. $y = -x^2 + 5$

26. $y = 2x^2 - 12$

27. $y = -2x^2 - 1$

28. $y = -3x^2 - 2$

29. $y = \frac{3}{4}x^2 - 3$

30. $y = \frac{1}{5}x^2 + 10$

31. $y = \frac{1}{2}x^2 - 5$

32. $y = -\frac{2}{3}x^2 + 9$

EXAMPLE 5
for Exs. 33–36

33. ★ **MULTIPLE CHOICE** How would the graph of the function $y = x^2 + 3$ be affected if the function were changed to $y = x^2 + 9$?

Ⓐ The graph would shift 9 units to the right.

Ⓑ The graph would shift 6 units up.

Ⓒ The graph would shift 9 units up.

Ⓓ The graph would shift 6 units down.

COMPARING GRAPHS **Tell how you can obtain the graph of *g* from the graph of *f* using transformations.**

34. $f(x) = x^2 - 5$
$g(x) = x^2 + 8$

35. $f(x) = 3x^2 - 11$
$g(x) = 3x^2 - 16$

36. $f(x) = 4x^2$
$g(x) = 2x^2$

CHALLENGE **Write a function of the form $y = ax^2 + c$ whose graph passes through the two given points.**

37. $(-1, 9), (0, 3)$

38. $(2, 1), (5, -20)$

39. $(-2, -16.5), (1, 4.5)$

PROBLEM SOLVING

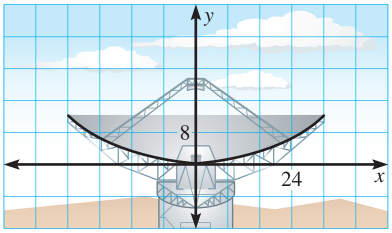
GRAPHING CALCULATOR **You may wish to use a graphing calculator to complete the following Problem Solving exercises.**

EXAMPLE 6
for Exs. 40–41

40. **ASTRONOMY** A cross section of the parabolic surface of the antenna shown can be modeled by the graph of the function $y = 0.012x^2$ where x and y are measured in meters.

a. Find the domain of the function in this situation.

b. Find the range of the function in this situation.



41. **SAILING** Sailors need to consider the speed of the wind when adjusting the sails on their boat. The force F (in pounds per square foot) on a sail when the wind is blowing perpendicular to the sail can be modeled by the function $F = 0.004v^2$ where v is the wind speed (in knots).

a. Graph the function for wind speeds from 0 knots to 50 knots.

b. Use the graph to estimate the wind speed that will produce a force of 1 pound per square foot on a sail.

c. Estimate the wind speed that will produce a force of 5 pounds per square foot on a sail.

REVIEW VERTICAL MOTION

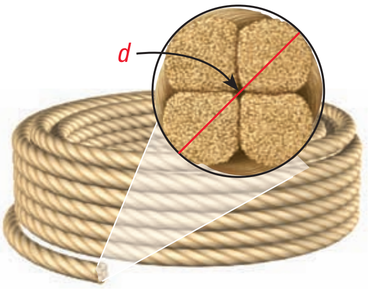
You may want to review the vertical motion model for Ex. 42.

42. FALLING OBJECTS Two acorns drop from an oak tree. One falls 45 feet, while the other falls 32 feet.

a. For each acorn, write an equation that gives the height h (in feet) of the acorn as a function of the time t (in seconds) it has fallen.

b. *Describe* how the graphs of the two equations are related.

43. ★ SHORT RESPONSE The breaking strength w (in pounds) of a manila rope can be modeled by the function $w = 8900d^2$ where d is the diameter (in inches) of the rope.



a. Graph the function.

b. If a manila rope has 4 times the breaking strength of another manila rope, does the rope have 4 times the diameter of the other rope? *Explain*.

44. ★ EXTENDED RESPONSE For an engineering contest, you have to create a container for an egg so that the container can be dropped from a height of 30 feet without breaking the egg.

a. The distance y (in feet) that the container falls is given by the function $y = 16t^2$ where t is the time (in seconds) the container has fallen. Graph the function.

b. The height y (in feet) of the dropped container is given by the function $y = -16t^2 + 30$ where t is the time (in seconds) since the container is dropped. Graph the function.

c. How are the graphs from part (a) and part (b) related? *Explain* how you can use each graph to find the number of seconds after which the container has fallen 10 feet.

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45. CHALLENGE The kinetic energy E (in joules) of an object in motion is given by $E = \frac{1}{2}mv^2$ where m is the object's mass (in kilograms) and v is the object's velocity (in meters per second). Suppose a baseball has 918.75 joules of energy when traveling 35 meters per second. Use this information to write and graph an equation that gives the energy E of the baseball as a function of its velocity v.